Streaky 3D structures in the Boundary Layer

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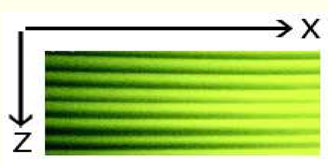
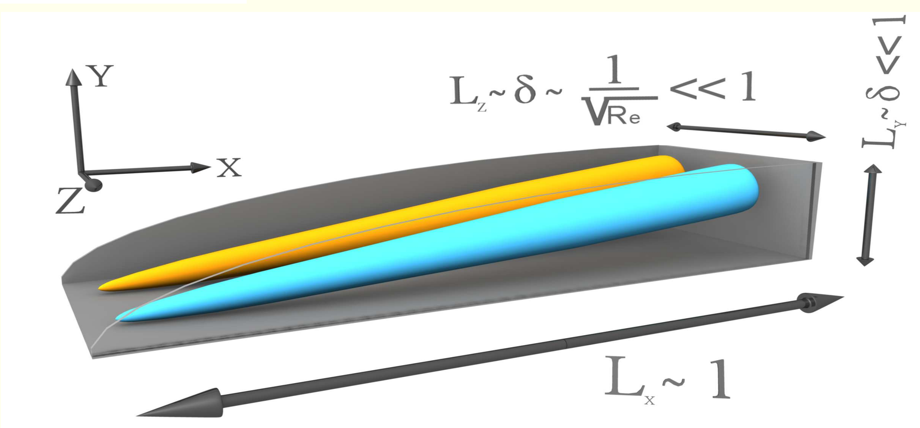
Introduction

3D Streaky Laminar Boundary Layer Flow



Smoke visualization of streaky flow, [1] [2].



Sketch of the different streak scales

Reduced Navier Stokes (RNS)

Steady Streak description for large Re

$$L_x \sim 1, \quad L_y, L_z \sim \frac{1}{\sqrt{Re}} << 1; \quad u \sim 1, \quad v, w \sim \frac{1}{\sqrt{Re}} << 1$$

3D Boundary layer scaling

$$x = \hat{x} \qquad u = \hat{u} + \ldots$$
$$y = \hat{y}\sqrt{Re} \quad v = \hat{v}\sqrt{Re} + \ldots$$
$$z = \hat{z}\sqrt{Re} \quad w = \hat{w}\sqrt{Re} + \ldots$$
$$p = \hat{\mathbf{p}}_0 + \frac{1}{Re}\hat{\mathbf{p}}_1 + \ldots$$

TWO TERMS for the pressure are required

$$\left.\begin{aligned} Y - Momentum \rightarrow \frac{\partial \hat{p}_0}{\partial \hat{y}} = 0 \\ Z - Momentum \rightarrow \frac{\partial \hat{p}_0}{\partial \hat{z}} = 0 \end{aligned}\right\} \rightarrow \hat{p}_0 = \hat{p}_0(\hat{x})$$

RNS equations (after dropping tildes)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{dp_0}{dx} + \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial p_1}{\partial y} + \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{\partial p_1}{\partial z} + \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

The 2^{nd} order y and z momentum eqs. are required to compute v and w, and the pressure correction term p_1 is now coupled.

- RNS have been used for high Re microchannel and microtube flow computations (see, e.g. [3], [4] and [5]).
- Never used before, up to our knowledge, for external boundary layer flow computations.

Standard 2D BL equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{dp_0}{dx} + \left(\frac{\partial^2 u}{\partial y^2}\right)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p_1}{\partial y} + \left(\frac{\partial^2 v}{\partial y^2}\right)$$

The 2^{nd} order y momentum eq. is only required now to compute the pressure correction p_1.

Numerical Method

Parabolic evolution in x.
Solving RNS in the $y - z$ plane for each station in x.

Discretization

- Simple one step Euler implicit in the stream-wise direction (x)
- Compact finite difference scheme in the wall-normal direction (y) - 2^{nd} order accuracy.
- Central difference scheme in the spanwise direction (z) - 2^{nd} order accuracy.

Method

- Decoupling (x) momentum eq. and $(y - z)$ momentum eqs.
- Sparse matrix solver for x marching.
- Speed improvement by constant matrix calculations.

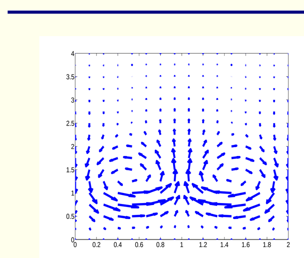
Conclusions

- RNS are derived from the complete Navier-Stokes for the description of large flow structures in the high Re limit.
- RNS formulation allows us to perform 3D streaky BL computations with much less CPU cost than standard 3D DNS.

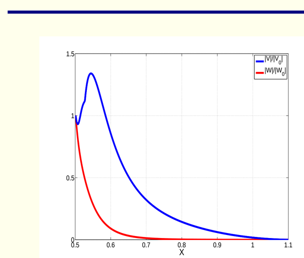
RESULTS: Natural Decay of an Initial Perturbation

Wall-normal and spanwise velocity

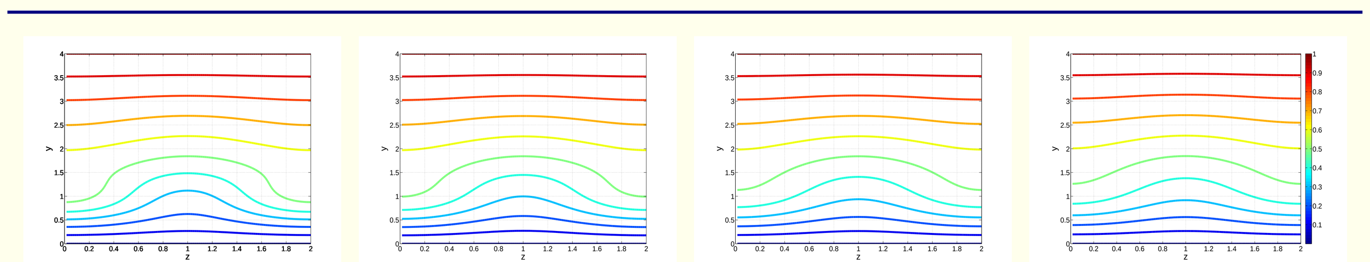
Initial



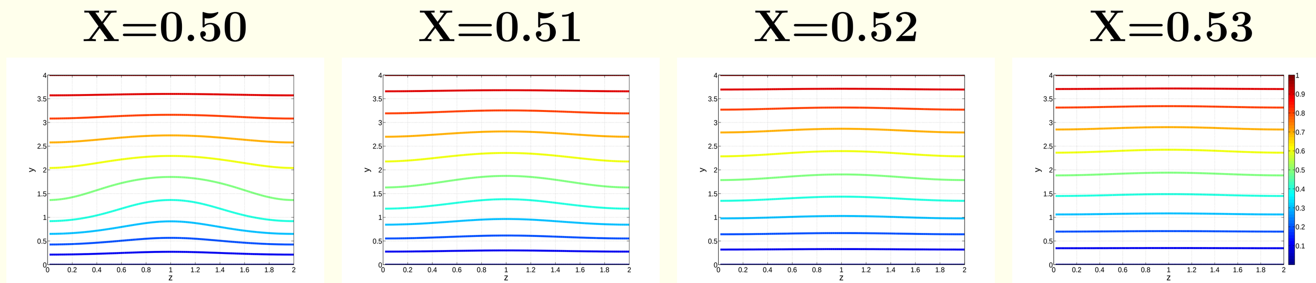
Evolution



Streamwise velocity contour plots

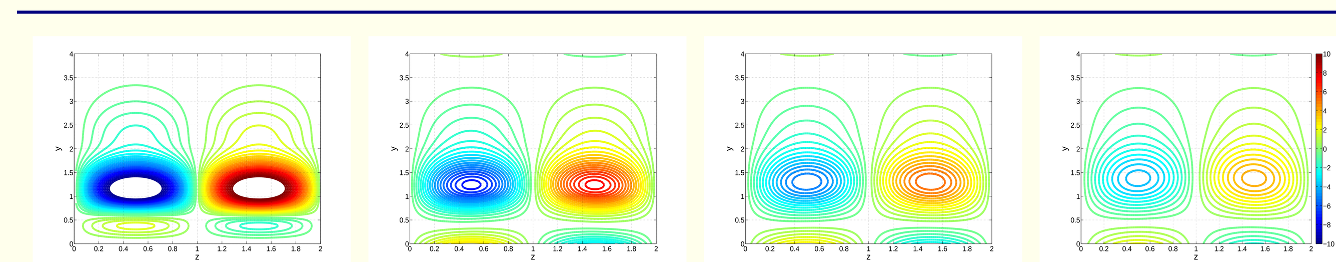


X=0.50 X=0.51 X=0.52 X=0.53

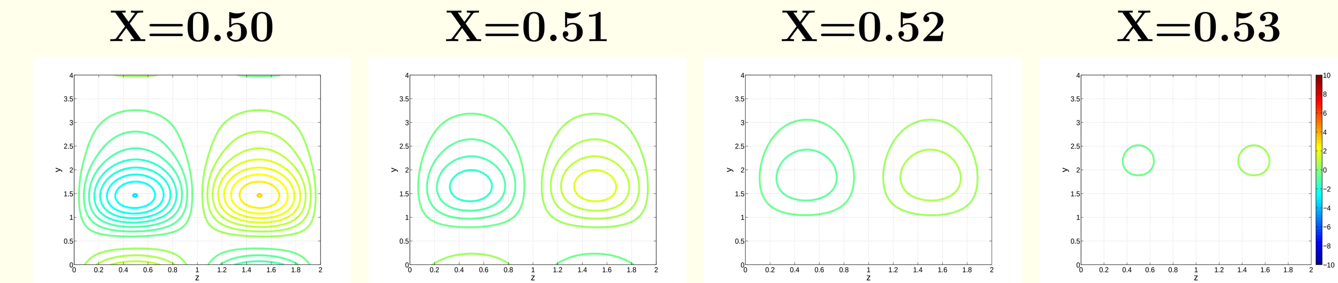


X=0.54 X=0.58 X=0.62 X=0.66

Streamwise vorticity contour plots



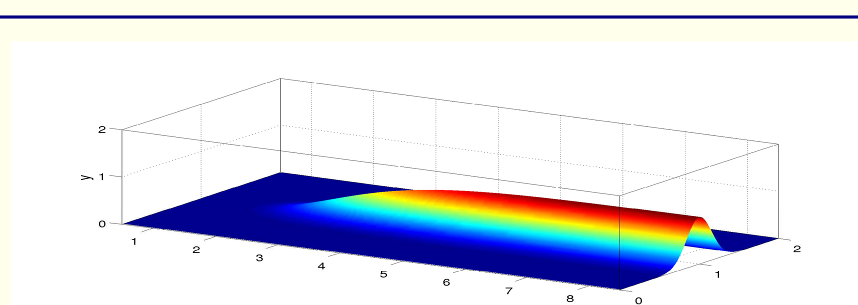
X=0.50 X=0.51 X=0.52 X=0.53



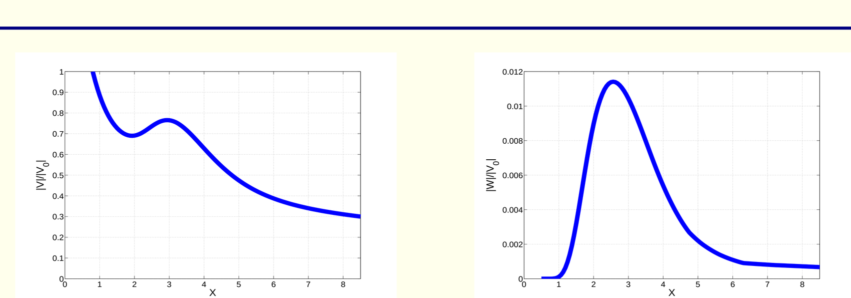
X=0.54 X=0.56 X=0.58 X=0.62

RESULTS: Streaks Generation Via Surface Roughness

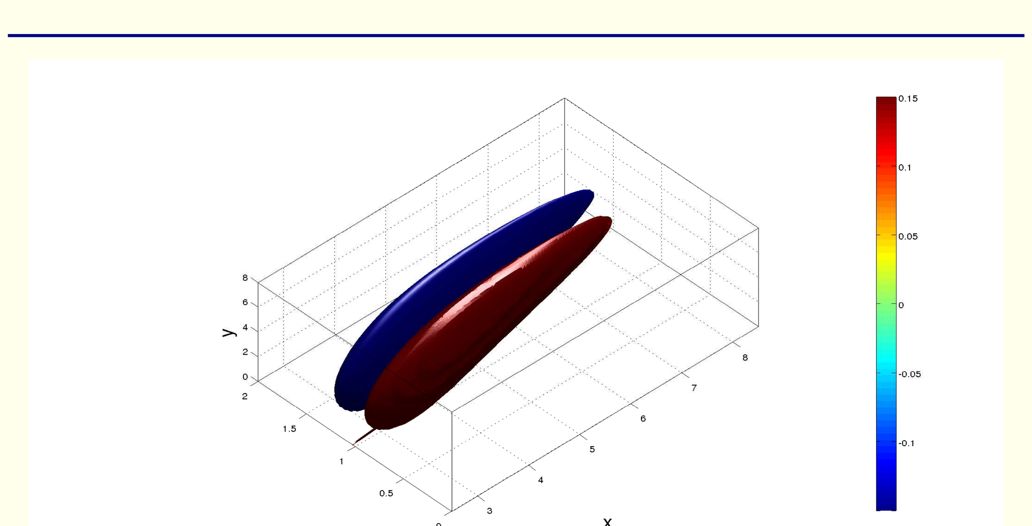
Surface perturbation



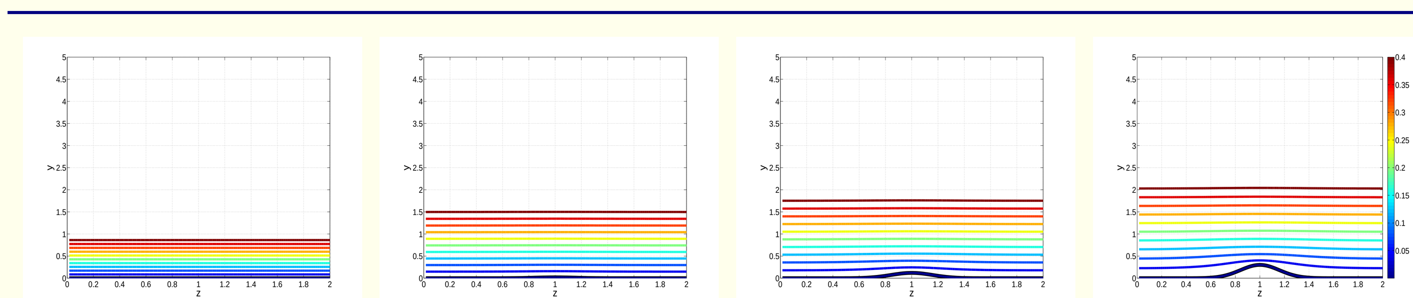
Wall-normal and spanwise velocity evolution.



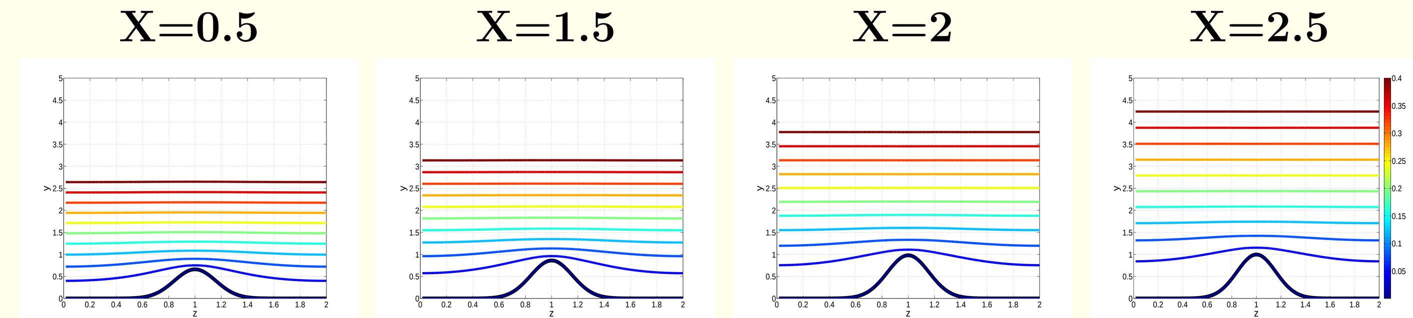
Streamwise vorticity isosurfaces



Streamwise velocity contour plots

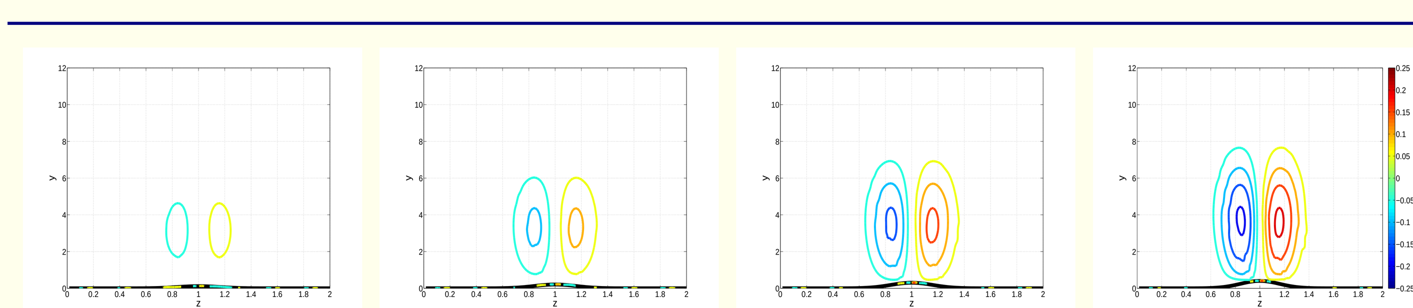


X=0.5 X=1.5 X=2 X=2.5

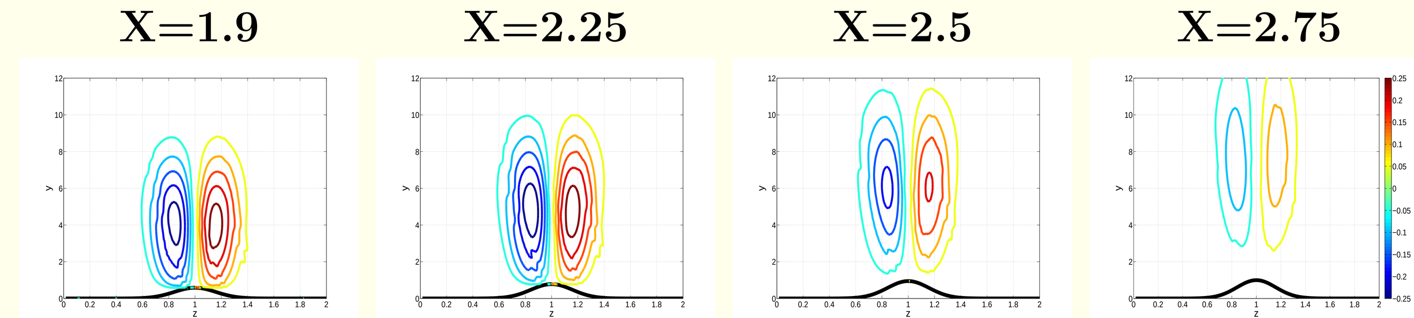


X=3.5 X=4.5 X=6.5 X=8.5

Streamwise vorticity contour plots

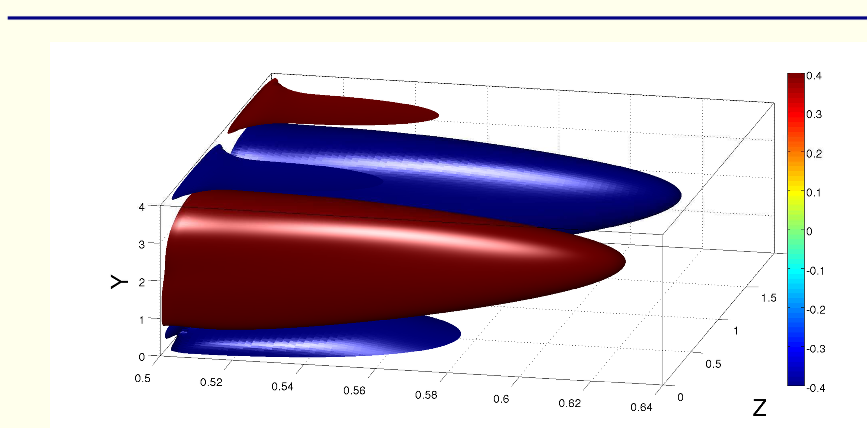


X=1.9 X=2.25 X=2.5 X=2.75



X=3.25 X=4 X=5.5 X=8.5

Streamwise Vorticity Isosurfaces



CPU cost

Computer	Box	Time
Intel Xeon Dual Core 8Gb RAM 3.0 GHz 64 bits architecture	$X = [0.5 - 12.5]$ 512x256x125 $(N_x \times N_y \times N_z)$	4800 seconds
Intel Xeon Core2 Duo 4Gb RAM 2.24 GHz 32 bits architecture	$X = [0.5 - 12.5]$ 256x128x125 $(N_x \times N_y \times N_z)$	1700 seconds

References

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[2] Choi, K.S. "The Rough with the Smooth", Nature, 2006.

[3] Chen, C.S. "Reduced Navier-Stokes Simulation of Incompressible Microchannels Flows", Numerical Heat Transfer, 2008.

[4] Chen, C.S. "Numerical Method for Predicting Three-Dimensional Steady Compressible Flow in Long Microchannels", J. Micromech. Microeng. 2004.

[5] Fletcher, C.A.J., "Computational Techniques for Fluid Dynamics, vol II" Springer-Verlag.

[6] Schlichting, H., "Boundary Layer Theory", McGraw-Hill.